Application of Rayleigh-Gans Formalism to Light Scattering by Platelets

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PROBLEM

Platelet function is correlated with platelet shape. The determination of the potency of stored platelets prior to transfusion is essential in transfusion therapy. The development of a technique for assessing the shape of platelets becomes more important as the shelf-life of platelets is extended to seven days.

OBJECTIVE

The objective is to examine the feasibility of using an optical technique to assess platelet shape by examining the scattering properties of platelets in the bag. The problem is complicated by the nonspherical shape of the platelets. The deviation of the particle shape from spherical increases the complexity of any computation. The Rayleigh-Gans formalism was used to compute the scattering from nonspherical platelets.

METHOD

The point spread function of a platelet solution is computed by propagating a laser beam through the platelet bag. The bag is assumed to be supported by a rigid frame which then fixes the optical path length a distance D = 2 cm as shown in Fig. 1. The resultant scattering pattern is computed for various solutions containing linear combinations of platelet shapes using a Monte Carlo technique. The usual assumption, that nonspherical particles can be replaced by "equivalent spheres" based on equal volume or equal surface area is not applicable in a geometry where multiple scattering is significant

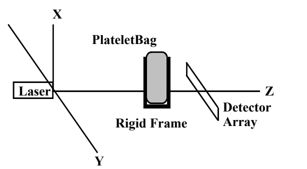

Fig. 1. A rigid frame is used to hold the platelet concentrate in position. Small holes in the frame allow the laser beam to enter and exit the bag. The point spread function is then measured using a detector array.

Optical Parameters

The attenuation coefficient of the platelet concentrate is given by

$$\beta = \beta^f + \beta^p \qquad (1)$$

where the attenuation due to the fluid may be further defined in terms of the fluid absorption and scattering coefficients β_A and β_S, respectively. The attenuation due to the platelets is given by

$$\beta^p = N(\sigma_A + \sigma_S) \qquad (2)$$

where N is the platelet number density and σ_A and σ_S are the absorption and scattering cross sections of the platelets, respectively. The scattering phase function defined by

$$\int_0^{2\pi}\int_0^{\pi} f(\theta_S)\sin\theta d\theta d\phi = \varpi = \sigma_S / (\sigma_A + \sigma_S) \qquad (3)$$

where the scattering angle θ_S may be defined in terms of the spherical angles θ and φ and ϖ is the single scattering albedo. The quantities σ_A, σ_S, and f are functions of particle size, shape, composition, and orientation. The Monte Carlo method uses random number generators to compute the path of the individual photons through the platelet solution. The individual photons are launched in the z direction. This path trajectory of each photon is then followed until one of the following conditions is met:

(1) the photon is incident on some element of the detector,
(2) the photon is deemed sufficiently outside the platelet volume so it is unlikely to reach the detector,
(3) the photon intensity is decreased to an insignificantly low value (method of partial photons) or absorbed.

Rayleigh- Gans Formalism

The tenets of the validity of the Rayleigh-Gans approximation are:

(1) $|m - 1| < 1$ where m is the index of refraction of the platelets **relative** to the medium. Hence, the platelet index of refraction is assumed very close to that of water (1.33).

(2) $4\pi r|m - 1| < \lambda$ where 2r is the characteristic dimension of the platelet.

The angular dependence of the scattered radiation is obtained by evaluating a form factor defined as

$$F(\theta,\phi) = \int_V \exp(i\delta)dV \qquad (4)$$

where $\delta = 2\pi\mathbf{R}\cdot(\mathbf{e}_i - \mathbf{e}_s)$ where **R** is the coordinate of element dV of the particle and **ei** and **es** are unit vectors in the initial and scattered directions. Phase functions of disks, spheroids, and spheres are shown in Fig 2.

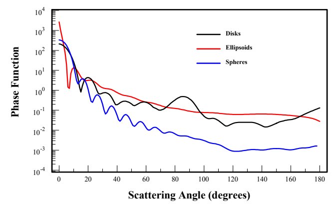

Fig. 2. Comparison of the phase functions of disk, spheroid, and spherical platelets.

Disks

The platelet absorption and scattering efficiencies are computed by using the Rayleigh-Gans formalism for a short circular cylinder (van de Hulst, 1981). The phase function was computed using Eq.(4) for typical platelet dimensions (radius = 1.55 µm, thickness = 1.0 µm) and averaged over all possible orientations, each of which were equally weighted.

Spheroids

An exact solution for scattering by spheroidal particles was developed by Asano and Yamamoto (1975). They used a separation of variables in the vector wave equation with a subsequent expansion in spheroidal wavefunctions. The phase function was obtained by evaluating the form factor (Eq. 4) for all possible orientations of a spheroid described by two radii of curvature. When the symmetry axes of the spheroid are oblique to the direction of the incident radiation, the scattering phase function is asymmetric. This feature is averaged out by considering all orientations of the spheroid.

Sphere

Mie theory (Mie, 1908) was used to compute the phase function and cross sections of spherical platelets. Spherical platelets are not perfectly spherical but have receptors and other appendages protruding from the spherical body. Approximations of the platelets as coated spheres were examined to determine if this approximation may be more realistic. The resultant was highly dependent on the index of refraction and thickness of the coating. In the absence of data on the physical validity of the approximation, the approach was abandoned. The phase function of a single particle exhibits a periodic structure due to surface waves.

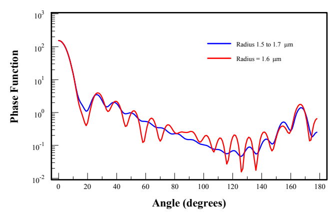

Fig. 3. Comparison of phase function of a sphere of radius 1.6 µm with the average phase function of a range of spheres between 1.5 µm and 1.7 µm.

This is demonstrated in Fig. 3 where the average phase function from spheres in a size range (radius = 1.5 µm to 1.7 µm) is compared with the phase function of a 1.6 µm sphere. Backscattering (scattering angle = 180°) can vary by two orders of magnitude over a small difference in radius.

Computation of the Optical Properties of the Platelet Concentrate

The optical properties of a distribution of platelets were computed from a linear combination of the three platelet shapes using the following equation;

$$\beta^p = \sum_i (N_i(\sigma_A^i + \sigma_S^i)) \qquad (5)$$

where the summation is over the three platelet shapes and f_i is the platelet shape fraction. Representative values of platelet dimensions were used in the simulations. The value of N was set equal to 10^{10} cm^{-3}.

RESULTS

Several theoretical distributions of platelets were examined using the Monte Carlo program. The resultant point spread functions for four distributions with a disk fraction ranging from 0.85 to 0.33 are shown in Fig. 4. The point spread function has been normalized with a peak value of 1.0. The point spread function is dependent on the phase function and cross section of the platelets. Because the number density of platelets in the bag will vary from bag to bag, the shape rather than the magnitude of the point spread function must be used to estimate the values of the f_i. A comparison of the integrated normalized intensity of the point spread function is shown in Fig. 5.

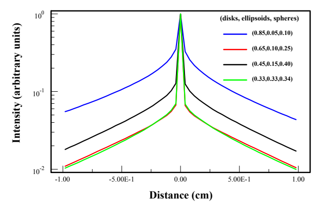

Fig. 4. Comparison of point spread functions for four linear combination of platelet shapes. The magnitude on the optical axis (Distance = 0.0) has been set to unity.

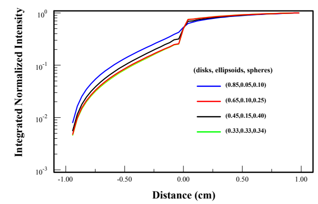

Fig. 5. Comparison of Integrated normalized intensity for four linear combination of platelet shapes. The curves shown in Fig. 4 were integrated and then normalized.

REFERENCES

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